THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

Geometry

Overview

The Common Core State Standards for Mathematics (CCSSM) were developed to influence increased rigor and coherence within mathematics classrooms across the United States (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The standards were developed using previous state standards and drew on input from various sources such as international curricula, education researchers, educators, parents, students, and other educational organizations. The CCSSM are designed to set expectations for deep understanding of mathematics by providing sequential standards that consider research based learning progressions across the grades. Furthermore, preparing students to succeed in college, career, and life.

The Standards for Mathematical Practice indicate mathematical habits that students should develop and use across the grades. They should become more sophisticated over time as they drive a deeper understanding of mathematics.

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Organization

Standards define what students should understand and be able to do.
Clusters are groups of standards.
Domains are larger groups of related standards.

Domain

Geometry 3.G

Reason with shapes and their attributes.

Standard

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Cluster

Key Shifts

The CCSSM were designed to build on previous state standards. Thus in order to understand their primary differences, the key shifts indicate the critical aspects of the CCSSM.

1) Greater focus on fewer topics.

2) Coherence: Linking topics and thinking across grades.

3) Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity.

Geometry

is the only content area with standards across all grade levels.

The CCSSM Geometry standards follow three streams aligned to learning progressions for geometric understanding.

Stream 1: Understand properties of geometric figures and the logical connections between them.

Stream 2: Develop and use formulas to compute lengths, areas and volumes.

Stream 3: Coordinate and analytic geometry (starting in Grade 5).

(Common Core Standards Writing Team, 2016).

Geometry

Across the Grades

The following includes the clusters (in bold) and a summary of the standards with the geometry domain for each grade.

Kindergarten

Identify and describe shapes.

Students are able to name and describe two and three dimensional shapes and objects regardless of their orientations or size.

Analyze, compare, create, and compose shapes.

Students are able to compare two and three dimensional shapes was well as model shapes in the world and compose complex shapes from simple ones.

Grade 1

Reason with shapes and their attributes.

Students are able to distinguish between defining and non-defining attributes of shapes. They are able to use this knowledge to create composite shapes. They are also able to partition circles and rectangles into equal parts and describe those parts using words like *half*, *fourth*, and *quarter*.

Grade 2

Reason with shapes and their attributes.

Students recognize and draw shapes based on specific attributes. They are able to partition rectangles into rows and columns of equal parts. They are able to partition circles and rectangles into multiple equal parts and describe them. Students recognize relationships among areas of shapes and their partitions.

Grade 3

Reason with shapes and their attributes.

Students recognize that shapes in various categories share attributes. Students recognize and draw different examples of quadrilaterals—rhombuses, rectangles and squares. Students express relationships among partitions of shapes in terms of fractions and whole units.

Grade 4

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Students draw lines and angles as two dimensional figures including parallel and perpendicular lines. They classify two-dimensional figures based on their lines and angles or based on the presence or absence of parallel or perpendicular lines. Students recognize and identify right triangles. Students can define, identify, draw, and use lines of symmetry.

Grade 5

Graph points on the coordinate plane to solve real-world and mathematical problems.

Students describe and apply conventions for graphing points on a two-dimensional coordinate plane. Students represent and interpret points on a coordinate plane in a real-world context.

Classify two-dimensional figures into categories based on their properties.

Students understand that two-dimensional figures with the same properties all belong to the same category and their sub-categories. Students classify figures based on the hierarchy of their properties.

Grade 6

Solve real-world and mathematical problems involving area, surface area, and volume.

Students explore and find the area of triangles, special quadrilaterals, and polygons. Students explore the concept of and formula for finding volumes of rectangular prisms. Students find the lengths of polygon sides in the coordinate plane. Students represent three-dimensional figures using nets, and apply these techniques to real-world problems.

Grade 7

Draw, construct and describe geometrical figures and describe the relationships between them.

Students solve problems involving scale figures. Students draw geometric shapes with a focus on constructing triangles from three measures of sides or angles, and determining which information determines a unique triangle, multiple triangles, or none. They describe two-dimensional figures from sections of three-dimensional figures.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

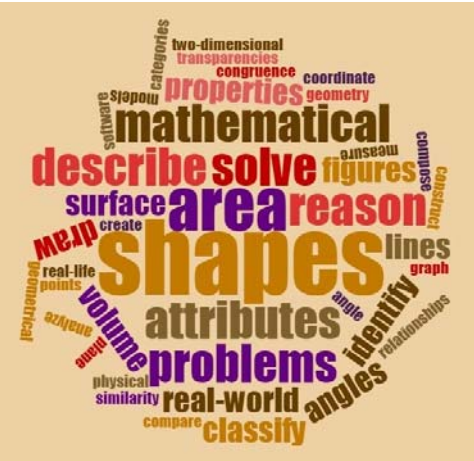
Students solve problems involving area and circumference of circles. Students use facts about angles in order to write and solve equations to find angle measures. They solve real-world problems involving area, volume, and surface area.

Grade 8

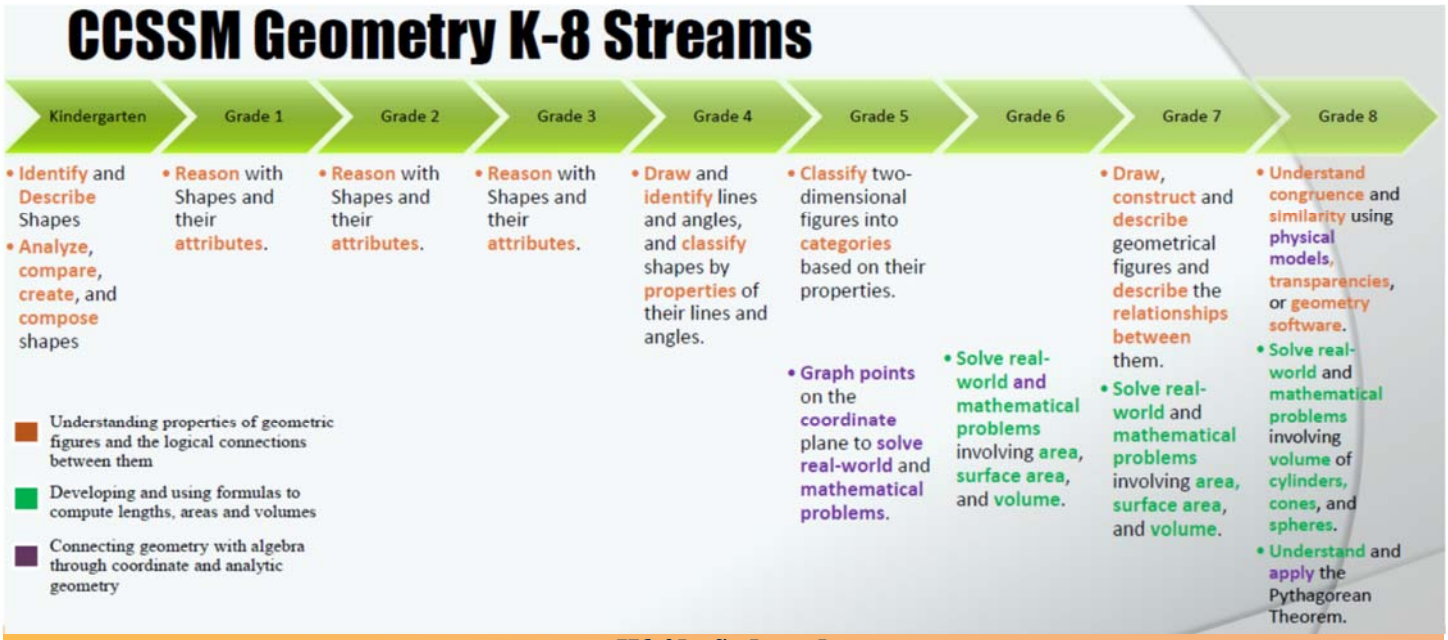
Understand congruence and similarity using physical models, transparencies, or geometry software.
Students verify experimentally transformations of lines and angles and understand similarity and congruence in terms of transformations. Students use informal arguments to explain relationships among angles of triangles and parallel lines and transversals.

Understand and apply the Pythagorean Theorem.
Students explain a proof of the Pythagorean Theorem and its converse and apply them in a coordinate system and real-world problems.

Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Students use volume formulas to find the volume of cones, cylinders, and spheres.



Word cloud created from K-8 Geometry.



High School

Congruence

Experiment with transformations in the plane.
Students use and develop precise definitions about rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Students experiment with transformations using various tools such as graph paper, tracing paper, or geometry software and describe geometric transformations in terms of functions and sequences.

Understand congruence in terms of rigid motions.
Students use rigid motions to transform figures, and use the definition of congruence in terms of rigid motions to explore congruent triangle relationships and criteria.

Prove geometric theorems.
Students prove theorems about lines, angles, triangles, and parallelograms.

Make geometric constructions.
Students make geometric constructions with a variety of tools such as compass and straightedge and dynamic geometric software. Students construct and copy angles, lines, bisectors, and figures inscribed in circles.

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.
Students explore similarity through dilations in the plane and use transformations to determine similarity of figures and determine the proportionality of all corresponding pairs of sides. They use similarity transformations to develop the AA criterion for two triangles to be similar.

Prove theorems involving similarity.
Students prove theorems about triangles. They use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.
Students use similarity to understand right triangle trigonometric ratios. They use trigonometric ratios to solve real-world right triangle problems.

Apply trigonometry to general triangles.
Students derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of triangles. They prove the Laws of Sines and Cosines and use them to solve real-world problems.

Circles

Understand and apply theorems about circles.
Students prove that all circles are similar. They identify and describe relationships among inscribed angles, radii, and chords. Students construct inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle. They construct a tangent line from a point outside the circle.
Find arc lengths and areas of sectors of circles.
Students derive and apply definitions about arcs, radian measures, and the formula for the area of a sector.

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.
Students derive the equations of circles, parabolas, ellipses, and hyperbolas.
Use coordinates to prove simple geometric theorems algebraically.
Students use coordinates to prove geometric theorems algebraically. Students prove the slope criterion for parallel and perpendicular lines. Students use coordinate geometry to find points on given line segments, and find perimeter and area of polygons.

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.
Students give informal arguments for circumference and area of circles, and volume formulas. Students use volume formulas to solve problems.
Visualize relationships between two-dimensional and three-dimensional objects.
Students identify shapes of two-dimensional cross sections of three-dimensional objects, and the three-dimensional shapes formed by rotations of two-dimensional objects.

Modeling with Geometry

Apply geometric concepts in modeling situations.
Students use geometric shapes, their measures, and their properties to describe objects. Students apply concepts of density based on area and volume in modeling situations and apply geometric methods to

Proof in Geometry

Prove geometric theorems

- Prove theorems about lines and angles:
  - *vertical angles are congruent;*
  - *when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;*
  - *points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- Prove theorems about triangles:
  - *measures of interior angles of a triangle sum to 180°;*
  - *base angles of isosceles triangles are congruent;*
  - *the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;*
  - *the medians of a triangle meet at a point.*
- Prove theorems about parallelograms:
  - *opposite sides are congruent,*
  - *opposite angles are congruent,*
  - *the diagonals of a parallelogram bisect each other, and conversely,*
  - *rectangles are parallelograms with congruent diagonals.*

Prove theorems involving similarity

- Prove theorems about triangles:
  - *a line parallel to one side of a triangle divides the other two proportionally, and conversely;*
  - *the Pythagorean Theorem proved using triangle similarity.*
- Use congruence and similarity criteria for triangles to prove relationships in geometric figures.

Apply trigonometry to general triangles

- Prove the Laws of Sines and Cosines.

Understand and apply theorems about circles

- Prove that all circles are similar.
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Use coordinates to prove simple geometric theorems algebraically

- Use coordinates to prove simple geometric theorems algebraically.
- Prove the slope criteria for parallel and perpendicular lines.

Geometric Constructions

Students complete the following constructions:

- Copying a segment
- Copying an angle
- Bisecting a segment
- Bisecting an angle
- Constructing perpendicular lines
- Perpendicular bisector of a line segment
- A line parallel to a given line through a point not on the line

- An equilateral triangle
- A square
- A regular hexagon inscribed in a circle
- The inscribed circles of a triangle
- The circumscribed circles of a triangle
- A tangent line from a point outside a given circle to the circle

van Hiele Levels

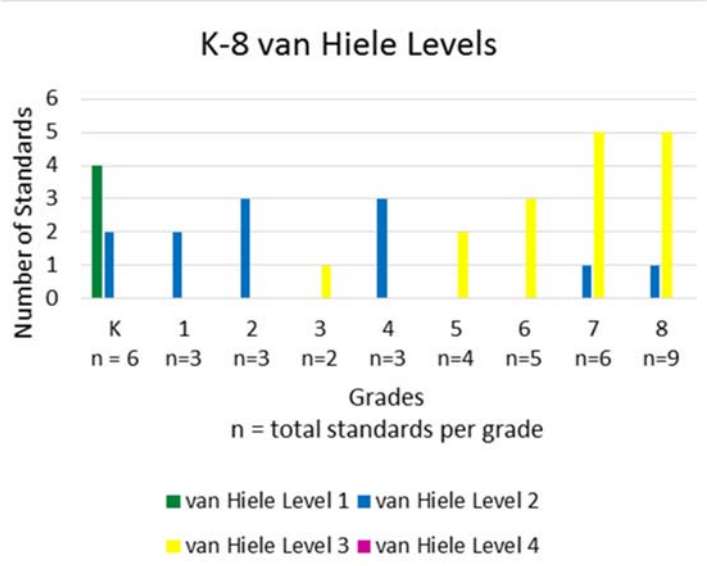
van Hiele Levels in CCSSM Geometry

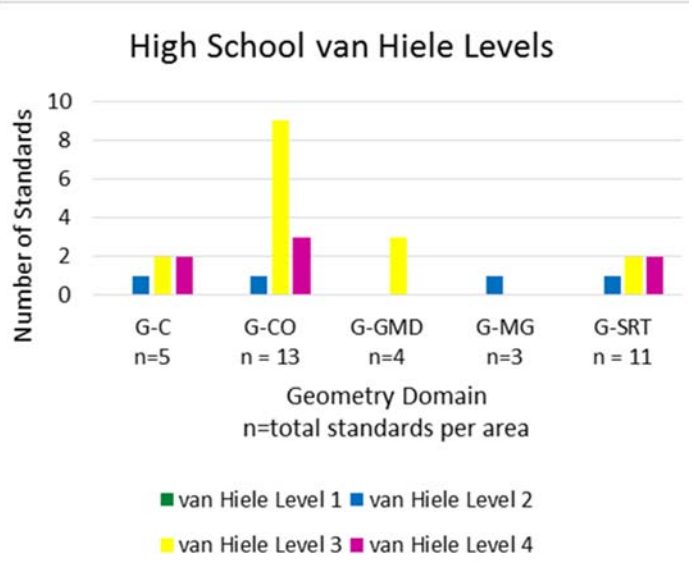
Developed by Dina van Hiele-Geldoff and Pierre Marie van Hiele in order to understand more about students' understanding of geometry (as cited in Senk, 1989), the van Hiele levels progress in difficulty. Researchers have investigated how students' van Hiele levels of understanding may predict student achievement in writing proofs (Senk, 1989; Usiskin, 1982), and they have found strong correlations. That is, higher van Hiele levels indicated higher achievement in proof writing. Thus in order for students to excel in high school geometry and beyond, students need to develop foundational understanding in earlier grades. Using a framework developed from multiple sources (Bassarear, 2008; Burger & Shaughnessy, 1986; Crowley, 1987; Newton, 2011; Senk, 1983; Senk, 1989; Van de Walle, Karp & Bay-Williams, 2007; & Wang & Kinzel, 2014), an analysis of CCSSM Geometry revealed which van Hiele levels are emphasized in each grade.

Summary of the van Hiele Levels and Their Characteristics

Levels	Characteristics	Examples
1 Visualization	Students identify, name, compare, and operate on geometric figures according to their appearance.	K.G.A.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects...
2 Analysis	Students analyze figures in terms of their components and relationships among components and discover properties and rules of a class of shapes empirically.	2.G.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces...
3 Informal Deduction	Students logically interrelate previously discovered properties and rules by giving or following informal arguments.	8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles...
4 Deduction	Students prove theorems deductively and establish interrelationships among networks of theorems.	HSG.CO.9 Prove theorems about lines and angles….HSG.CO.11 Prove theorems about parallelograms...
5 Rigor	Students establish theorems in different postulational systems (i.e. non-Euclidean) and analyze and compare these systems.	No standards address level 5.

The table above was a summary from Cirillo (2009, p. 252).





Notes:

*None of the standards addressed van Hiele level five.

*Standards addressing Stream 3 were not analyzed (e.g. coordinate geometry and trigonometry).

G-C = Circles

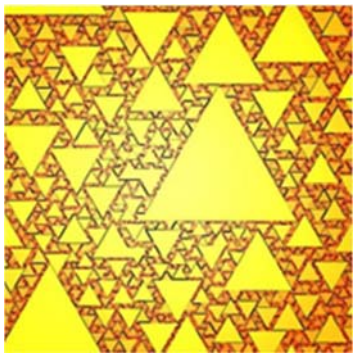
G-CO = Congruence

G-GMD = Geometric Measurement & Dimension

G-MG = Modeling with Geometry

G-SRT = Similarity, Right Triangles, and Trigonometry

Resources



Common Core State Standards Initiative
http://www.corestandards.org/Math/
Common Core Math Standards Progressions Documents
http://ime.math.arizona.edu/progressions/

Books:

Chval, K. B., Lannin, J. K., & Jones, D. (2016). *Putting Essential Understanding of Geometry and Measurement into Practice in grades 3-5*. NCTM.

Ellis, A. B., Bieda, K., & Knuth, E. J. (2012). *Developing Essential Understanding of Proof and Proving for Teaching Mathematics in grades 9-12*. NCTM.

Goldenberg, P., Clements, D. (2014). *Developing Essential Understanding of Geometry and Measurement for Teaching Mathematics in Pre-K-Grade 2*. NCTM.

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Wang, S., & Kinzel, M. (2014). How do they know it is a parallelogram? Analysing geometric discourse at van Hiele Level 3. *Research in Mathematics Education*, *16*(3), 288–305.

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