Common Core Geometry Proof – Triangles_3

Midsegment Theorem

Theorem: If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half the length.

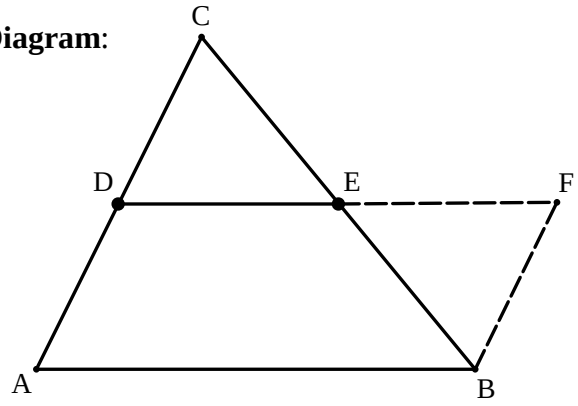
Given: $\overline{DE}$ where D is the midpoint of $\overline{AC}$ and E is the midpoint of $\overline{CB}$

Construction: Construct $\overline{DF}$ where E is the midpoint, and draw $\overline{BF}$

Prove: $\overline{DE} \parallel \overline{AB}$

$DE = \frac{1}{2}AB$

Diagram:



Statements	Reasons
1. $\overline{DE}$ where D is the midpoint of $\overline{AC}$ and E is the midpoint of $\overline{CB}$	1. Given
2. Construct $\overline{DF}$ where E is the midpoint, and draw $\overline{BF}$	2. Construction
3. $\overline{AD} \cong \overline{DC}$ and $\overline{BE} \cong \overline{EC}$	3. Definition of Midpoint
4. ∠CED and ∠BEF are vertical angles	4. Definition of Vertical Angles
5. ∠CED ≅ ∠BEF	5. Theorem: If two angles are vertical angles, then they are congruent
6. △DCE ≅ △FBE	6. SAS ≅ SAS
7. $\overline{DC} \cong \overline{FB}$	7. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8. $\overline{AD} \cong \overline{FB}$	8. Transitive Axiom
9. ∠DCE ≅ ∠FBE	9. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
10. ∠DCE and ∠FBE are alternate interior angles	10. Definition of Alternate Interior Angles
11. $\overline{DA} \parallel \overline{FB}$	11. Theorem: If alternate interior angles formed by lines cut by a transversal are congruent, then the lines cut by the transversal are parallel.
12. $\overline{DE} \parallel \overline{AB}$	12. Theorem: If a quadrilateral has two opposite sides parallel and congruent to each other, then the other two opposite

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13. DE + EF = DF	sides of the quadrilateral are parallel and congruent to each other.
14. $\overline{DE} \cong \overline{EF}$	13. Partition Postulate
15. DE = EF	14. Definition of Midpoint
16. $\overline{AB} \cong \overline{DF}$	15. Definition of Congruent
17. AB = DF	16. Theorem from line12
18. DE + EF = AB	17. Definition of Congruent
19. DE + DE = AB	18. Substitution Axiom
20. 2DE = AB	19. Substitution Axiom
21. DE = $\frac{1}{2}$ AB	20. Addition Axiom
	21. Division Axiom